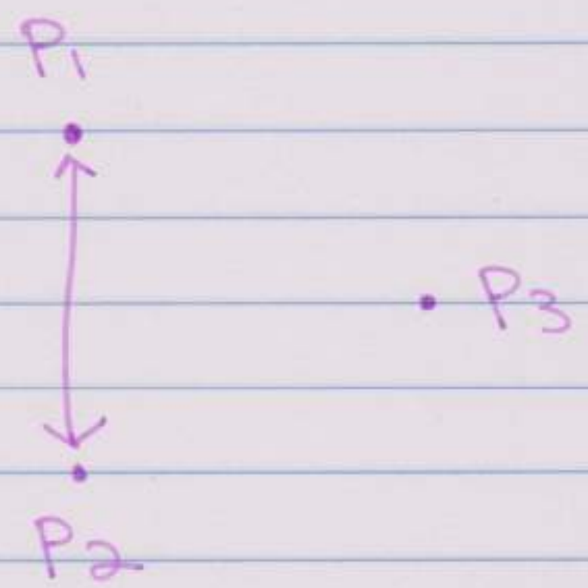
Solar System Example

Consider a solar system with 2019 planets s.t. the pairwise distances of the planets are distinct. Suppose there is a person on each planet and all of them are observing their closest planet. Show that there is a planet that is not observed by any person.

Soln:

Suppose there are 3 planets, P_1, P_2 and P_3 s.t. P_1 and P_2 have the smallest pairwise distance. Then, the person on P_1 will be observing P_2 and the person on P_2 will be observing P_1 and nobody is observing P_3.



Now, suppose there are 2019 planets and P_1 and P_2 have the smallest pairwise distance. Now, you have 2 cases:

Case 1: If there is someone observing either P_1 or P_2 and is from one of the other 2017 planets, then we are done.

Case 2: If nobody is observing P_1 or P_2, then problem reduces to 2017 planets. Eventually, either Case 1 will be used or the problem reduces to 3 planets, in which case I have already explained why there is a planet that is not observed.